# STUDY ON ENERGY ANALYSIS OF HOPFIELD NETWORK

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**Abstract;**

The energy landscape is determined only by the network architecture, i.e., the number of units, their output functions, threshold values, connections between units and the strengths of the connections. Hopfield has shown that for symmetric weights with no self-feedback, i.e. $w_{ij} = w_{ji}$, and with bipolar {-1, +1} or binary output functions, the dynamics of the network using the asynchronous update always leads towards energy minima at equilibrium. The states corresponding to these energy minima turn out to be a stable state, which means that small perturbations around it lead to unstable states. Hence the dynamics of the network takes the network back to a stable state again. It is the existence of these stable states that enables us to store patterns, one at each of these states.

**Key words:** number of units, their output functions, threshold values, connections between units.

## INTRODUCTION:

Automatic (machine) recognition, description, classification, and grouping of patterns are important problems in a variety of engineering and scientific disciplines such as biology, psychology, medicine, marketing, computer vision, artificial intelligence, and remote sensing. But what is a pattern Watanabe [2] defines a pattern as opposite of a chaos; it is an entity, vaguely defined, that could be given a name. For example, a pattern could be a fingerprint image, a handwritten cursive word, a human face, or a speech signal. Given a pattern, its recognition/classification may consist of one of the following two tasks : (1) supervised classification (e.g., discriminant analysis) in which the input pattern is identified as a member of a predefined class, (2) unsupervised classification (e.g., clustering) in which the pattern is assigned to a hitherto unknown class. Note that the recognition problem here is being posed as a classification or categorization task, where the classes are either defined by the system designer (in supervised classification) or are learned based on the similarity of patterns (in unsupervised classification). Interest in the area of pattern recognition has been renewed recently due to emerging applications which are not only challenging but also computationally more demanding. These applications include data mining (identifying a pattern, e.g., correlation, or an outlier in millions of multidimensional patterns), document classification (efficiently searching text documents), financial forecasting, organization and retrieval of multimedia databases, and biometrics (personal identification based on various physical attributes such as face and fingerprints).

## REVIEW OF LITERATURE:

Whitley [3] has identified a novel application of pattern recognition, called affective computing which will give a computer the ability to recognize and express emotions, to respond intelligently to human emotion, and to employ mechanisms of emotion that contribute to rational decision making. A common characteristic of a number of these applications is that the available features (typically, in the thousands) are not usually suggested by domain experts, but must be extracted and optimized by data-driven procedures.

The design of a pattern recognition system essentially involves the following three aspects:

1) data acquisition and preprocessing, 2) data representation, and 3) decision making. The problem domain dictates the choice of sensor(s), preprocessing technique, representation scheme, and the decision making model. It is generally agreed that a well-defined and sufficiently constrained recognition problem (small intra-class variations and large interclass variations) will lead to a compact pattern representation and a simple decision making strategy. Learning from a set of examples (training set) is an important and desired attribute of most pattern recognition systems. The four best known approaches for pattern recognition are: 1) template matching, 2) statistical classification, 3) syntactic or structural matching, and 4) neural networks. These models are not necessarily independent and sometimes the same pattern recognition method exists with different interpretations. Attempts have been made to design hybrid systems involving multiple models.

## MATERIAL AND METHOD:

Associated with each state of the network, Hopfield proposed an energy function whose values always either reduces or remains the same as the state of the network changes. Assuming the threshold value of the unit *i* to be $\theta_i$ , the energy function is given by [33]

$$V(s) = V = -\frac{1}{2}\sum_{i}\sum_{j} w_{ij} s_i s_j + \sum_{i} \theta_i s_i$$

The energy *V(s)* as a function of the state **s** of the network describes the energy landscape in the state space.

Let us consider the change of state due to update of one unit, say *k*, at some instant. All other units remain unchanged. We can write the expressions for energy before and after the change as follows:

$$V^{old} = -\frac{1}{2}\sum_{i}\sum_{j} w_{ij} s_{i_i}^{old} s_j^{old} + \sum_{i} \theta_i s_i^{old}$$

$$V^{new} = -\frac{1}{2}\sum_{i}\sum_{j} w_{ij} s_{i_i}^{new} s_j^{new} + \sum_{i} \theta_i s_i^{new}$$

The change in energy due to update of the *k*th unit is given by

$$\Delta V = V^{new} - V^{old}$$

$$= -\frac{1}{2}\sum_{i\neq k}\sum_{j\neq k} w_{ij}\left(s_i^{new} s_j^{new} - s_{i_i}^{old} s_j^{old}\right) + \sum_{i\neq k} \theta_i \left(s_i^{new} - s_{i_i}^{old}\right)$$

$$= -\frac{1}{2}\sum_{i} w_{ik} s_{i_i}^{new} s_k^{new} - \frac{1}{2}\sum_{j} w_{kj} s_{k_i}^{new} s_j^{new} + \theta_k s_{k_i}^{new}$$

$$-\frac{1}{2}\sum_{i} w_{ik} s_{i_i}^{new} s_k^{new} - \frac{1}{2}\sum_{j} w_{kj} s_{k_i}^{new} s_j^{new} + \theta_k s_{k_i}^{new}$$

$$+\frac{1}{2}\sum_{i} w_{ik} s_{i_i}^{old} s_k^{old} + \frac{1}{2}\sum_{j} w_{kj} s_{k_i}^{old} s_j^{old} + \theta_k s_{k_i}^{old} \qquad (3.14)$$

Since $s_{i_i}^{new} = s_{i_i}^{old}$ , for $i \neq k$ , the first two terms on the right hand side of Eq. (3.14) will be zero. Hence,

$$\Delta V = s_{k_i}^{new}\left[-\frac{1}{2}\sum_i w_{ik} s_{i_i}^{new} - \frac{1}{2}\sum_j w_{kj} s_j^{new} + \theta_k\right]$$

$$+ s_k^{old}\left[+\frac{1}{2}\sum_i w_{ik} s_{i_i}^{old} + \frac{1}{2}\sum_j w_{kj} s_j^{old} - \theta_k\right] \qquad (3.15)$$

If the weights are assumed symmetric, i.e., $w_{ij} = w_{ji}$, then we get

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$$\Delta V = -s_{k_i}^{new}\left[\sum_i w_{ki} s_{i_i}^{new} - \theta_k\right] + s_k^{old}\left[\sum_i w_{ki} s_{i_i}^{old} - \theta_k\right] \qquad (3.16)$$

If in addition, $w_{kk} = 0$, then since $s_{i_i}^{new} = s_{i_i}^{old}$ for $i \neq k$, the terms in both the parentheses are equal. Therefore,

$$\Delta V = \left(s_k^{old} - s_k^{new}\right)\left[\sum_i w_{ki} s_{i_i}^{old} - \theta_k\right] \qquad (3.17)$$

The update rule for each unit *k* is as follows:

Case A: If $\sum_i w_{ki} s_{i_i}^{old} - \theta_k > 0$, then $s_{k_i}^{new} = +1$

Case B: If $\sum_i w_{ki} s_{i_i}^{old} - \theta_k < 0$, then $s_{k_i}^{new} = -1$

Case C: If $\sum_i w_{ki} s_{i_i}^{old} - \theta_k = 0$, then $s_{k_i}^{new} = s_{k_i}^{old}$

For case A, if $s_{k_i}^{old} = +1$, then $\Delta V = 0$, and if $s_{k_i}^{old} = -1$, then $\Delta V \leq 0$.

For case B, if $s_{k_i}^{old} = +1$, then $\Delta V < 0$, and if $s_{k_i}^{old} = -1$, then $\Delta V = 0$.

For case C, irrespective of the value of $s_{k_i}^{old}$, $\Delta V = 0$.

Thus we have $\Delta V \leq 0$. Therefore the energy decreases or remains the same when a unit, selected at random, is updated provided the weights are symmetric, and the self-feedback is zero. This is the energy analysis for discrete Hopfield model.

**CONCLUSION**

Till now, we are through with preliminaries of pattern recognition, neural networks, genetic algorithms and one specific model of ANN i.e. Hopfield model by and large used for storing (memorizing the patterns). In the next chapter, we will try to focus our discussion on formation of hybrid evolutionary systems by combining the genetic algorithms with Hopfield model of ANN. This combination, when applied to different pattern recognition problems, gives better results compared to ANN or GA alone.

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